The notation is slightly different from the original book.

1 Issue I: Definition of $\|\cdot\|_\infty$ Not Explicit

Page 70, the proof of **Theorem 3.12**, we used the infinity norm of a continuous function on the display

$$\begin{aligned} &2\|\varphi\|_\infty \mathbb{P}_x\big\{\tau\big(\partial\mathcal{B}_\delta(z)\big) < \tau\big(C_z(\alpha)\big)\big\} + \varepsilon\mathbb{P}_x\big\{\tau(\partial U) < \tau\big(\partial\mathcal{B}_\delta(z)\big)\big\} \\ &\le 2\|\varphi\|_\infty a^k + \varepsilon. \end{aligned}$$

We know that the infinity norm is of the form

$$\|\varphi\|_\infty := \sup_{x\in\partial U} |\varphi(x)|,$$

but there was no explicit description of $\|\cdot\|_\infty$.

2 Issue II: Justification of an Integral Estimate in the Dvoretzky–Erdős Theorem

Page 75, in the proof of the Dvoretzky-Erdös test **Theorem 3.22**, we have

$$\begin{aligned} \mathbb{E}_0\int_0^\infty 1_{\{|B_s|\le\rho\}}ds &= \rho^2\int_0^\infty \mathbb{P}\{|B_s|\le 1\}ds \\ &\le \rho^2\left(1+\int_1^\infty \frac{\mathcal{L}\big(\mathcal{B}_1(0)\big)}{(2\pi s)^{d/2}}ds\right) = C_4\rho^2, \end{aligned}$$

the first equality holds by Brownian scaling; as for the second inequality, for it to be true, we need the inequality

$$\int_0^\infty \mathbb{P}\{|B_s|\le 1\}ds \le 1+\int_1^\infty \frac{\mathcal{L}\big(\mathcal{B}_1(0)\big)}{(2\pi s)^{d/2}}ds$$

to be true. However, there is no explicit reference to the validity. In fact, in the next section, **Theorem 3.26** tells us that this is true since the occupation measure μ_t is absolutely continuous with respect to the Lebesgue measure.

3 Issue III: Ambiguity in the Domain of the Dirichlet Problem

Page 69, the original statement of the Dirichlet problem **Theorem 3.12**

$$\text{Let } \tau := \inf\{t>0 : B_t\in\partial U\}, \text{ which is an almost surely finite stopping time.}$$

This caused confusions as if the Brownian motion does not start inside ∂U, it may happen that $\tau(\partial U)$ is infinite. We should explicitly state that the Brownian motion starts inside ∂U.

4 Issue IV: Typo in the Definition of A_n^*

Page 133, in the proof of **Lemma 5.24**, the definition of A_n^* was originally

$$\begin{aligned} A_n^* := &\left\{\text{there exists } t \in [0,1) \text{ such that } \left|W_n\big(T_k/n)\big) - W_n(t)\right| > \varepsilon\right\} \\ &\cup \left\{\text{there exists } t \in [0,1) \text{ such that } \left|W_n(T_{k-1}/n) - W_n(t)\right| > \varepsilon\right\}, \end{aligned}$$

there is a typo labeled in red.

5 Issue V: Indexing Error in Summation

Page 293, in the proof of the upper bound for **Theorem 10.3**, the last inequality was stated as

$$\sum_{k=m}^{\infty} \sum_{j=0}^{\eta^k - 1} \eta^{-k\gamma} \mathbb{P}\left\{ \frac{|B_{j\eta^{-k}+k\eta^{-k}} - B_{j\eta^{-k}}|}{\sqrt{2k\eta^{-k}\log\left(\frac{\eta^k}{k}\right)}} > a(1-4\varepsilon)\right\} \leq \sum_{k=1}^{\infty} \eta^k \eta^{-k\gamma} \eta^{-ka^2(1-4\varepsilon)^3} < \infty,$$

where the red term $\eta^k - 1$ should be $\lceil \eta^k - 1 \rceil$.

6 Issue VI: Ambiguity Arises from a Classical Trick in Fractal Geometry

Page 300, in the proof for the upper bound of **Theorem 10.15**. The author thought it was a mistake, but after taking a semester-long fractal geometry course the author realizes it is in fact true. The original context is still confusing for people without much knowledge in fractal geometry (Chapter 4 may not suffice).

We first demonstrate why it is confusing:

Theorem 6.1 (Theorem 10.15)**.** *For every metric space E we have*

$$\dim_P(E) = \inf\left\{ \sup_{i=1}^{\infty} \overline{\dim}_M(E_i) : E \subseteq \bigcup_{i=1}^{\infty} E_i, E_i \text{ bounded}\right\}. \tag{6.1}$$

The idea of the proof is that: first we prove

$$P(A, 4\varepsilon) \leq M(A, 2\varepsilon) \leq P(A, \varepsilon). \tag{6.2}$$

Then we can use (6.2) to prove the lower bound in (6.1) where we used a common trick in fractal geometry: let $s > \dim_P(E)$ be arbitrary and once we can show that $\overline{\dim}_M(E) \leq t$, we can let $s \downarrow \dim_P(E)$ and conclude the result.

Having this trick in mind, in the proof for the upper bound, our choice is that

$$0 < t < s < \dim_P(E), \tag{6.3}$$

and $A_i \subset E$ are bounded such that $E = \bigcup_{i=1}^{\infty} A_i$. The claim is

$$\text{It suffices to show that } \overline{\dim}_M(A_i) \geq t \text{ for some } i. \tag{6.4}$$

Consider the same trick and (6.4), it does not suffice to show (6.1) since

$$\overline{\dim}_M(A_i) \geq t \text{ for some } i \not\Rightarrow \overline{\dim}_M(E) \geq s,$$

as $s < \dim_P(E)$ is arbitrary, sending $s \uparrow \dim_P(E)$ does not conclude the proof.

However, (6.4) suffices to conclude the proof, it is not s but t being arbitrary. This should be trivial via the graph of packing dimension:

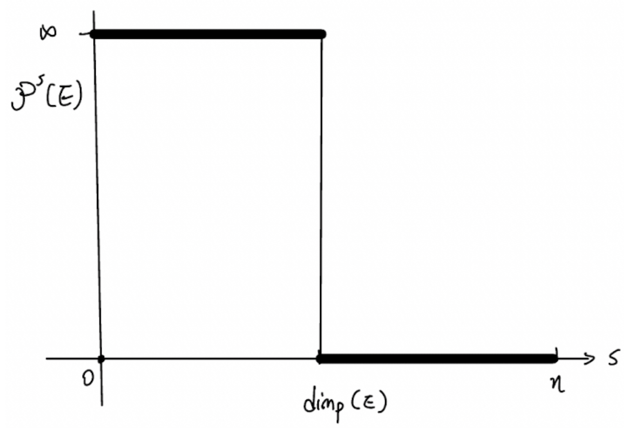

Figure 1: Graph of $\mathcal{P}^s(E)$ against a set E. The packing dimension is the value of s at which the 'jump' from ∞ to 0 occurs.

7 Issue VII: Incorrect Section Reference

Page 301, the original statement is:

> Before moving back to our study of Brownian paths we study the packing dimension of the 'test sets' we have used in the stochastic co-dimension method in Section 9.9.1.

Apparently there is no **Section 9.9.1**, and the stochastic co-dimension method was introduced in **Section 9.1.2**.

8 Issue VIII: Typo in Expectation Expression

Page 321, in the proof of **Lemma 10.47**, in the first display we have

$$\begin{aligned}
&\mathbb{E}\big[Z(I)Z(J)1_{\{T_{\delta/2}(z_I)<T_{\delta/2}(z_J)\}}v\big] \\
&\leq \mathbb{E}\Bigg[1_{\Big\{B\big(0,T_\delta(Z_I)\big)\subset z_I+W[\alpha,\pi]\Big\}} \times \mathbb{E}_{B\big(T_{\delta/2}(z_I)\big)}\Big[1_{\Big\{B\big(0,S^{(0)}_{\eta/2}(z_I)\big)\subset z_I+W[\alpha,\pi]\Big\}} \\
&\qquad \times \mathbb{E}_{B\big(T_{\eta/2}(z_J)\big)}\Big[1_{\Big\{B\big(0,T_\delta(z_J)\big)\subset z_J+W[\alpha,\pi\Big\}} \\
&\qquad \times \mathbb{P}_{B\big(T_{\delta/2}(z_J)\big)}\Big\{B\big(0,S^{(0)}_{r_k}(z_J)\big)\subset z_J+W[\alpha,\pi]\Big\}\Big]\Big]\Bigg] \\
&\leq C^4\big(\frac{\delta}{|z_I|}\big)^{\pi/\alpha}\big(\frac{\delta}{\eta}\big)^{2\pi/\alpha}\big(\frac{2\delta}{R}\big)^{\pi/\alpha} \\
&\leq C_2|I|^{4\pi/\alpha}\mathrm{dist}(I,J)^{-2\pi/\alpha},
\end{aligned}$$

there is a typo v labeled in red which should not appear in the literature.

9 Issue IX: Minor Word Error

Page 322, after the proof of **Theorem 10.38**, before the statement of Adelman, the original text was

> A surprising consequence of the non-existence of cone points for angle smaller then $\pi\cdots$.

The word labeled in red should be replaced by than.