

1.

Page 70. In the proof of **Theorem 3.12**, we used the infinity norm of a continuous function in the display

$$\begin{aligned} & 2\|\varphi\|_\infty \mathbb{P}_x\{\tau(\partial\mathcal{B}_\delta(z)) < \tau(C_z(\alpha))\} + \varepsilon \mathbb{P}_x\{\tau(\partial U) < \tau(\partial\mathcal{B}_\delta(z))\} \\ & \leq 2\|\varphi\|_\infty a^k + \varepsilon. \end{aligned}$$

However, the definition of $\|\varphi\|_\infty$ was not given anywhere in the book, therefore it is natural to add the usual convention

$$\|\varphi\|_\infty := \sup_{x \in \partial U} |\varphi(x)|,$$

to achieve the maximum self-containedness.

2.

Page 75, last paragraph of the proof of Theorem 3.22. We have

$$\begin{aligned} \mathbb{E}_0 \int_0^\infty 1_{\{|B_s| \leq \rho\}} ds &= \rho^2 \int_0^\infty \mathbb{P}\{|B_s| \leq 1\} ds \\ &\leq \rho^2 \left(1 + \int_1^\infty \frac{\mathcal{L}(\mathcal{B}_1(0))}{(2\pi s)^{d/2}} ds \right) = C_4 \rho^2, \end{aligned}$$

where the first equality holds by Brownian scaling, but the justification of the second inequality relies on the inequality

$$\int_0^\infty \mathbb{P}\{|B_s| \leq 1\} ds \leq 1 + \int_1^\infty \frac{\mathcal{L}(\mathcal{B}_1(0))}{(2\pi s)^{d/2}} ds,$$

where it is not identified prior to **Theorem 3.22**, nor left any reference to the later discussion (we know it is due to Green's identity but we do not have an explicit expression). The citation should go after **Theorem 3.26**.

3.

Page 69. In the assertion of **Theorem 3.12**, the original statement “Let $\tau(\partial U) := \inf\{t > 0 : B_t \in \partial U\}$, which is an almost surely finite stopping time”. We should explicitly declare that $\{B_t\}_{t \geq 0}$ is a Brownian motion starting inside the domain, as otherwise this stopping time cannot be almost surely finite, especially when $d \leq 2$.

4.

Page 133. In the Proof of **Lemma 5.24**, the definition of A_n^* , which was originally stated as

$$\begin{aligned} A_n^* &:= \left\{ \text{there exists } t \in [0,1) \text{ such that } |W_n(T_k/n) - W_n(t)| > \varepsilon \right\} \\ &\cup \left\{ \text{there exists } t \in [0,1) \text{ such that } |W_n(T_{k-1}/n) - W_n(t)| > \varepsilon \right\}. \end{aligned}$$

There is a mistyped closed paranthesis labeled in red.

$$A_n^* := \left\{ \text{there exists } t \in [0,1) \text{ such that } |W_n(T_k/n) - W_n(t)| > \varepsilon \right\} \cup \left\{ \text{there exists } t \in [0,1) \text{ such that } |W_n(T_{k-1}/n) - W_n(t)| > \varepsilon \right\}$$

5.

Page 293. In the proof of upper bound for **Theorem 10.3**, the last inequalities are stated as

$$\begin{aligned}
& \sum_{k=m}^{\infty} \sum_{j=0}^{\lceil \eta^k - 1 \rceil} \eta^{-k\gamma} \mathbb{P} \left\{ \frac{|B_{j\eta^{-k} + k\eta^{-k}} - B_{j\eta^{-k}}|}{\sqrt{2k\eta^{-k} \log\left(\frac{\eta^k}{k}\right)}} > a(1 - 4\epsilon) \right\} \\
& \leq \sum_{k=1}^{\infty} \eta^k \eta^{-k\gamma} \eta^{-ka^2(1-4\epsilon)^3} \\
& < \infty
\end{aligned}$$

where the red term was given by $\eta^k - 1$, inconsistent with the previous equations.

6.

Page 300. In the proof of upper bound for **Theorem 10.15**, suppose

$$0 < t < s < \dim_P(E),$$

and let $\{A_i\}_{i=1}^{\infty}$ be a collection of bounded sets such that $E = \bigcup_{i=1}^{\infty} A_i$. The assertion that

It suffices to show that $\overline{\dim}_M(A_i) \geq t$ for some i

concludes the proof that

$$\dim_P(E) \leq \inf \left\{ \sup_{i=1}^{\infty} \overline{\dim}_M(A_i) : E = \bigcup_{i=1}^{\infty} A_i, A_i \text{ bounded} \right\}$$

does not hold. Having $\overline{\dim}_M(A_i) \geq t$ for some i will tell us that $\sup_{i=1}^{\infty} \overline{\dim}_M(A_i) \geq t$ and

thus $\inf \left\{ \sup_{i=1}^{\infty} \overline{\dim}_M(A_i) : E = \bigcup_{i=1}^{\infty} A_i, A_i \text{ bounded} \right\} \geq t$. But even this holds for every

t , we can only conclude that

$$t \leq \overline{\dim}_M(E)$$

but we have no further information for the comparison between $\overline{\dim}_M(E)$ and s . We need further information to guarantee that

$$s \leq \overline{\dim}_M(E)$$

and then sending $s \uparrow \dim_P(E)$ gives the desired result. But the above inequality is not justified in the proof.

7.

Page 301. “Before moving back to our study of Brownian paths we study the packing dimension of the ‘test sets’ we have used in the stochastic co-dimension method in **Section 9.9.1**.” There is no **Section 9.9.1** and the stochastic co-dimension method is introduced in **Section 9.1.2**. This is a clear typo.

8.

Page 321. In the proof of **Lemma 10.47**, in the first display we have

$$\begin{aligned}
& \mathbb{E} \left[Z(I)Z(J)1_{\{T_{\delta/2}(z_I) < T_{\delta/2}(z_J)\}} \textcolor{red}{v} \right] \\
& \leq \mathbb{E} \left[1_{\left\{ B(0, T_{\delta}(z_I)) \subset z_I + W[\alpha, \pi] \right\}} \times \mathbb{E}_{B(T_{\delta/2}(z_I))} \left[1_{\left\{ B(0, S_{\eta/2}^{(0)}(z_I)) \subset z_I + W[\alpha, \pi] \right\}} \right. \right. \\
& \quad \times \mathbb{E}_{B(T_{\eta/2}(z_J))} \left[1_{\left\{ B(0, T_{\delta}(z_J)) \subset z_J + W[\alpha, \pi] \right\}} \right. \\
& \quad \left. \left. \times \mathbb{P}_{B(T_{\delta/2}(z_J))} \left\{ B(0, S_{r_k}^{(0)}(z_J)) \subset z_J + W[\alpha, \pi] \right\} \right] \right] \right] \\
& \leq C^4 \left(\frac{\delta}{|z_I|} \right)^{\pi/\alpha} \left(\frac{\delta}{\eta} \right)^{2\pi/\alpha} \left(\frac{2\delta}{R} \right)^{\pi/\alpha} \\
& \leq C_2 |I|^{4\pi/\alpha} \text{dist}(I, J)^{-2\pi/\alpha}
\end{aligned}$$

where there is a typo labeled in red in the first identity, the term $\textcolor{red}{v}$ should be removed.

9.

Page 322. After the proof of Theorem 10.38, before the statement of Adelman, there is a typo in the original sentence

A surprising consequence of the non-existence of cone points for angles smaller $\textcolor{red}{then}$ $\pi \cdots$
There is a typo in the red term and should be replaced by “than”.