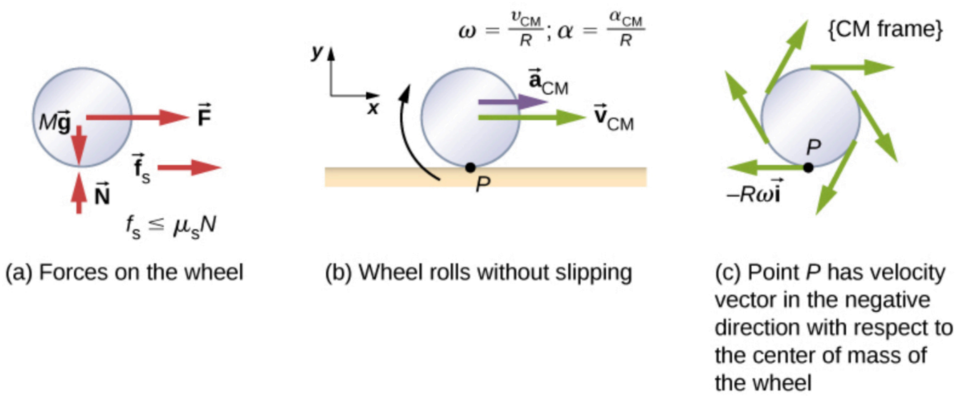
## Physics Lec 8

People have observed rolling motion without slipping ever since the invention of the wheel. For example, we can look at the interaction of a car's tires and the surface of the road. If the driver depresses the accelerator to the floor, such that the tires spin without the car moving forward, there must be kinetic friction between the wheels and the surface of the road. If the driver depresses the accelerator slowly, causing the car to move forward, then the tires roll without slipping. It is surprising to most people that, in fact, the bottom of the wheel is at rest with respect to the ground, indicating there must be static friction between the tires and the road surface. The tires have contact with the road surface, and, even though they are rolling, the bottoms of the tires deform slightly, do not slip, and are at rest with respect to the road surface for a measurable amount of time. There must be static friction between the tire and the road surface for this to be so.




(Figure 1.1)

If one considers the point where the wheel has a connection (that is, tangential) to the surface it is moving, denoting that point as $P$, then the distance, velocity, and the acceleration at point $P$ could be expressed as in (b) of Figure 1.1. Indeed, as for the velocity, it is trivial that one has

$$v_P = -R\omega\hat{i} + v_{CM}\hat{i}.$$

Since the velocity of $P$ relative to the surface is zero, we have $v_P = 0$ and it follows that $v_{CM} = R\omega$, differentiating both sides with respect to time $t$, one then has

$$a_{CM} = R\alpha.$$

Similarly, we can find the distance the wheel travels in terms of angular variables. As the wheel rolls from the bottom point $A$ to the top point $B$, its outer surface maps onto the ground by exactly the distance travelled, which we denote as $d_{CM}$. Since the length of the outer surface that maps onto the ground is the arc length $R\theta$, one then has the distance function being

$$d_{CM} = R\theta.$$

Now we shall offer an example, which complicates the problem involving a block slides down from an inclined ramp, where we the block is replaced by a ball.

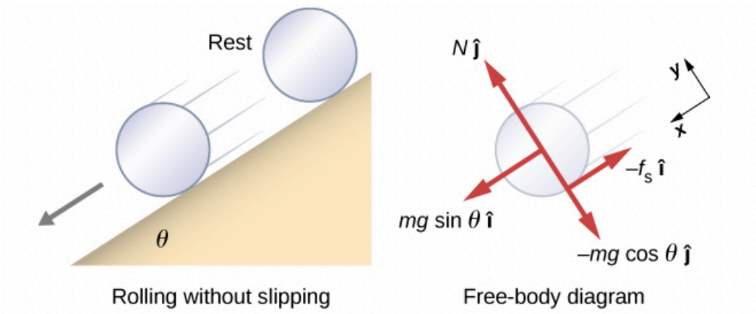
**Example 1.1**:

A solid cylinder rolls down an inclined plane without slipping, starting from rest. It has mass $m$ and radius $r$. What is its acceleration?

**Solution**:

As we did for problems involving Newton's Second law of force, we divide the solution into three steps.

*Step I*: Free-body Diagram




*Step II*: Applying Newton's Second Law of Force

For the net force we have two components, the $x$-direction and the $y$-direction, hence we have the net force being

$$\sum F_x = ma_x \text{ and } \sum F_y = ma_y,$$

where we know the net force is the direct sum of these two vectors.

Applying Newton's Second Law of Force yields

$$mg\sin\theta - f_s = m(a_{CM})_x \text{ and } N - mg\cos\theta = 0,$$

we can then solve for the linear acceleration of the center of mass from these equations:

$$(a_{CM})_x = g\sin\theta - \frac{f_s}{m}.$$

The third step offers an intuition that, the expression of the linear acceleration in terms of the moment of inertia is helpful in solving problems involving circular motions.

*Step III*: Applying Fixed-Rotation Results

Since $\sum \tau_{CM} = I_{CM}\alpha$ and the torques are calculated about the axis through the center of mass of the cylinder. The only nonzero torque is provided by the friction force. We have

$$f_s r = I_{CM}\alpha,$$

since $(a_{CM})_x = r\alpha$, substitution yields

$$f_s = \frac{I_{CM}\alpha}{r} = \frac{I_{CM}a_{CM}}{r^2},$$

where we then see

$$a_{CM} = g\sin\theta - \frac{I_{CM}a_{CM}}{mr^2} = \frac{mg\sin\theta}{m + (I_{CM}/r^2)}.$$

Note that this result is independent of the coefficient of static friction $\mu_s$. Since we have a solid cylinder then $I_{CM} = \frac{mr^2}{2}$ and

$$a_{CM} = \frac{mg\sin\theta}{m + \left(mr^2/2r^2\right)} = \frac{2}{3}g\sin\theta.$$

Therefore one has $\alpha = \frac{a_{CM}}{r} = \frac{2}{3r}g\sin\theta.$ ||

In solving this example, we see that the crucial idea is to apply the results from the fixed-rotation, that is:

$$a_{CM} = \frac{mg\sin\theta}{m + \left(I_{CM}/r^2\right)}.$$

It turns out this is a very useful expression for solving problems analogous to this.

So far we have only considered the case where the ball does not slide down from the ramp. If we consider the case when it slides, then we are encoutering little difference. Since, in this case, $v_{CM} \neq R\omega$ since the point where the ball connects to the surface is changing during the motion, that is, $v_P \neq 0$, $\omega \neq \frac{v_{CM}}{R}$, nor $\alpha = \frac{a_{CM}}{R}$.