Lecture Notes Week 3
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Abstract:
In this chapter we are going to set up the notations and elementary terminologies for later use. We introduce the scope and scale of physics in the first subsection, then for every scope and scale we are able to assign units, and we can also derive a conversion between any units. We proceed the discussion of dimension analysis in the third subsection. Then we introduce the estimates and Fermi Calculations in the fourth part. Lastly we introduce the significant figures, where we can describe the accuracy and errors of an experiment.

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1. Review On One-Dimension Case

Last time we introduced the motion in 1, 2, and 3 dimensions, and Newton's three laws. We now do a recap.

Definition: Displacement

Displacement is the change in position of an object, denoted by Δx, defined by $\Delta x = x_f - x_0$, where x_f represents the final position while x_0 represents the initial position.

If we want to record the total replacement of a collections of displacements, it follows that the displacement is an additive function since it is defined by only the difference between the final position and the initial position. We therefore define the total displacement of a collection of displacements, namely $\{x_i\}_{i=1}^n$, by

$$\Delta x = \Delta x_{\text{Total}} := \sum_{i=1}^{n} \Delta x_i, \tag{1.1}$$

rivially by the additivity of the summation operation, we can extend to the case of a countable collection of displacements, namely, $\{x_i\}_{i=1}^\infty$.

If we want to represent the total distance a particle travels, we need to use the notion distance, instead of displacement, which is defined by taking the absolute values of the replacement, i.e.,

$$x = x_{\text{Total}} = \sum_{i=1}^{n} |\Delta x_i|. \tag{1.2}$$

Since the time taken to travel between two particles is called the elapsed time Δt, we can then calculate the average velocity $\bar{v}$.

Definition: Average Velocity

If x_1 and x_2 are the positions of a particle at times t_1 and t_2, respectively, then the average velocity, denoted by $\overline{v}$, is defined by $\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$.

Note that the average velocity does not require our distance function to be differentiable! To find the instantaneous velocity, however, we shall let $\Delta t \to 0$, which then, requires the differentiability.

Definition: Instantaneous Velocity

The instantaneous velocity of an object is the limit of the average velocity as the elapsed time approaches zero, or the derivative of x with respect to t, i.e. $v(t) = \frac{d}{dt}x(t)$.

Speed, on the other hand, is defined to be the scalar.

Definition: Average Speed

The average speed, denoted by $\overline{s}$, is defined to be $\overline{s} = \frac{x}{\Delta t}$.

Similarly, we can define the instantaneous speed by

Definition: Instantaneous Velocity

The instantanoue speed is the magnitude of the instantaneous velocity, namely, instantaneous velocity $:= |v(t)|$.

Applying the above calculations once more by replacing the nominator by velocity (resp. speed), then we have the same argument of accelations.

Definition: Average Acceleration

Average acceleration is the rate at which velocity changes, it is denoted by $\overline{a}$, and is defined to be $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$, where v_f and t_f are the final velocity and the final time (traveled), respectively, and v_0 and t_0 are the initial velocity and the initial time (traveled), respectively.

Definition: Instantaneous Accelaration

If $v(t)$ is differentiable with respect to t, then the instantaneous acceleration, denoted by $a(t)$, is defined to be $a(t) = \frac{d}{dt}v(t)$.

We shall later use t instead of Δt, use $x - x_0$ instead of Δx, and use $v - v_0$ instead of Δv. Furthermore, we shall always assume that the acceleration is constant!

1.1 Fundamental Results

Theorem 1.1:

We adapt the updated notations, and we have

(i) $x = x_0 + \overline{v}t$. $\left(\text{since } \overline{v} = \frac{x - x_0}{t} = \frac{\Delta x}{\Delta t}\right)$

Since we assumed the acceleration to be constant, then

(ii) $\overline{v} = \frac{v_0 + v}{2}$.

(iii) $v = v_0 + at$. $\left(\text{since } a = \frac{v - v_0}{t} = \frac{\Delta v}{\Delta t}\right)$

(iv) $x = x_0 + v_0 t + \frac{1}{2}at^2.$

(v) $v^2 = v_0^2 + 2a(x - x_0).$

(vi) $a = \frac{v - v_0^2}{2(x - x_0)}.$

Proof:

The first three results follow from their own arguments. We prove the last three.

(iv):

Since $v = v_0 + at$, we add v_0 to each side and divide each side by two, then

$$\frac{v + v_0}{2} = v_0 + \frac{1}{2}at,$$

since one has $\bar{v} = \frac{v + v_0}{2}$, it follows that $\bar{v} = v_0 + \frac{1}{2}at$, but we have $x = x_0 + \bar{v}t$, thus,

$$\frac{x - x_0}{t} = v_0 + \frac{1}{2}at,$$

rearranging yields

$$x = x_0 + v_0 t + \frac{1}{2}at^2.$$

(v):

Since $v = v_0 + at$, one has $t = \frac{v - v_0}{a}$ (provided $a \neq 0$), but since $\bar{v} = \frac{v_0 + v}{2}$ and $x = x_0 + \bar{v}t$, one has then

$$x = x_0 + \frac{v + v_0}{2} \cdot \frac{v - v_0}{a} = x_0 + \frac{v^2 - v_0^2}{2a},$$

rearranging yields

$$2a(x - x_0) = v_0^2 - v^2,$$

result follows.

(vi):

This result is only a restatement of (v).

□

In particular, if we consider the accelaration is given by the constant $g = 9.8\text{m/s}^2$, and we replace the argument of x to be the height y, it follows that we have

$$v = v_0 - gt, y = y_0 + v_0 t - \frac{1}{2}gt^2, \text{ and } v^2 = v_0^2 - 2g(y - y_0).$$

Theorem 1.2:

The velocity $v(t)$ could be obtained by $v(t) = \int a(t)dt + C_1$, provided $a(t)$ is integrable (Riemann), where C_1 is constant as a function of t.

Proof:

Since $a(t) := \frac{d}{dt}v(t)$, then, once $a(t)$ is integrable, $\int a(t)dt = v(t) + C_1$.

$\square$

Similarly, we have

Theorem 1.3:

$x(t) = \int v(t)dt + C_2$, provided $v(t)$ is integrable (Riemann), where C_2 is constant as a function of t.

2. Review on Multi-Dimensional Case

Definition: Position Vector

We consider $x = x(t)$, $y = y(t)$, and $z = z(t)$, where $x, y, z \in \mathbb{R}$ are real vectors. Then the position vector $r \in \mathbb{R}^3$, is defined by

$$r(t) = x(t) \oplus y(t) \oplus z(t).$$

Note that the arguments $x(t)$, $y(t)$, and $z(t)$ may produce scalars, in which case, the numerical addition of these three values is not valid, hence we use the vector space operation $\oplus$ to denote this case, where it forces the addition to be vector addition. To aviod ambiguition, we could also use

$$r(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}. \tag{2.1}$$

The definition for all the notations we have encountered so far are similar to one-dimensional case.

Definition: Displacement Vector

The displacement vector, denoted by Δr, is defined by $\Delta r = r(t_2) - r(t_1)$, where we assume $t_2 \geq t_1$ are time space arguments.

Brownian motion is closely linked to the normal distribution. Recall that a random variable X is normally distributed with mean μ and variance σ^2 if

$$\mathbb{P}(X > x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_x^\infty \exp\left\{ \frac{-(u-\mu)^2}{2\sigma^2} du \quad \forall x \in \mathbb{R}.\right.$$

Definition: Brownian Motion

A real-valued stochastic process $\{B(t) \,|\, t \geq 0\}$ is called a (linear) Brownian motion with start in $x \in \mathbb{R}$ if the following holds:

(i) $B(0) = x$.

(ii) The process has independent increments, i.e., for all times $0 \leq t_1 \leq t_2 \leq \cdots \leq t_n$, the increments $B(t_n) - B(t_{n-1})$, $B(t_{n-1}) - B(t_{n-2})$, $\cdots$, $B(t_2) - B(t_1)$ are independent random variables.

(iii) $\forall t \geq 0$ and $\forall h > 0$, the increments $B(t+h) - B(t)$ are normally distribued with expectation zero and variance h.

(iv) The function $t \mapsto B(t)$ is almost surely continuous, i.e., the probability $\mathbb{P}(\{x \,|\, x \text{ is where the mapping is not continuous}\}) = 0$.

We say that $\{B(t) \,|\, t \geq 0\}$ is a standard Brownian motion if $x = 0$.

Example 2.1: Brownian Motion

If we denote the increments of $\{B(t) \,|\, t \geq 0\}$ to be

$$\Delta r_1 = 2 \oplus 1 \oplus 3, \Delta r_2 = -1 \oplus 0 \oplus 3,$$
$$\Delta r_3 = 4 \oplus -2 \oplus 1, \text{ and } \Delta r_4 = -3 \oplus 1 \oplus 2.$$

Find the total displacement of the particle from the origin.

Solution:

We form the sum of the displacements and add them as vectors:

$$\begin{aligned}\Delta r_{\text{Total}} &= \sum_{i=1}^{4} \Delta r_i \\ &= \left(2-1+4-3\right) \oplus \left(1+0-2+1\right) \oplus (3+3+1+2) \\ &= 2 \oplus 0 \oplus 9.\end{aligned}$$

The total displacement is given by the norm of Δr_{Total}, namely,

$$\|\Delta r_{\text{Total}}\| = \sqrt{2^2+0^2+9^2} = 9.2. \qquad \|$$

Definition: Velocity Vector

The velocity vector, denoted by $v(t)$, is defined to be

$$v(t) = \lim_{\Delta t \to 0} \frac{r(t+\Delta t) - r(t)}{\Delta t} = \frac{dr}{dt},$$

where $v(t) = v_x(t) \oplus v_y(t) \oplus v_z(t)$, with $v_x(t) = \dfrac{dx(t)}{dt}$, $v_y(t) = \dfrac{dy(t)}{dt}$, and $v_z(t) = \dfrac{dz(t)}{dt}$.

Definition: Average Velocity

The average velocity, denoted by $v_{\text{avg}} = \dfrac{r(t_2) - r(t_1)}{t_2 - t_1}$.

Definition: Acceleration Vector

The acceleration vector, denoted by $a(t)$, is defined to be

$$a(t) = \lim_{t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t} = \frac{dv(t)}{dt},$$

where $a(t) = a_x(t) \oplus a_y(t) \oplus a_z(t)$, with $a_x(t) = \dfrac{dv_x(t)}{dt}$, $a_y(t) = \dfrac{dv_y(t)}{dt}$, and $a_z(t) = \dfrac{dv_z(t)}{dt}$.

2.1 Fundamental Results

Example 2.2:

Suppose a skier is moving with an acceleration of 2.1m/s^2 down a slope of 15° at $t = 0$. With the origin of the coordinate system at the front of the lodge, her initial position and velocity are

$$r(0) = (75\hat{\imath} - 50\hat{\jmath})\text{m}$$

and

$$v(0) = (4.1\hat{\imath} - 1.1\hat{\jmath})\text{m/s}.$$

(a) What are the x-components and y-components of the skier's position and velocity as functions of time?

(b) What is her position at $t = 10$s?

Solution:

(a):

One has, according to the trigonometric result,

$$a_x = (2.1)\text{m/s}^2 \cos(15^\circ) = 2\text{m/s}^2,$$
and
$$a_y = -(2.1)\text{m/s}^2 \sin(15^\circ) = -0.54\text{m/s}^2,$$
where the negation is applied since the y-component is negative.
Since $v_x(t) = 4.1\text{m/s}$ and $a_x = 2\text{m/s}^2$, one has
$$\begin{aligned} x(t) &= x_0 + v_x t + \frac{1}{2}a_x t^2 \\ &= 75\text{m}+(4.1\text{m/s})t + \frac{1}{2}(2\text{m/s}^2)t^2, \end{aligned}$$
and
$$v_x(t) = 4.1\text{m/s}+(2.0\text{m/s}^2)t.$$
Similarly, since $v_y(t) = -1.1\text{m/s}$ and $a_y(t) = -0.54\text{m/s}^2$, one has
$$\begin{aligned} y(t) &= y_0 + v_y t + \frac{1}{2}a_y t^2 \\ &= -50\text{m}+(-1.1\text{m/s})t + \frac{1}{2}(-0.54\text{m/s}^2)t^2, \end{aligned}$$
and
$$v_y(t) = -1.1\text{m/s}+(-0.54\text{m/s}^2)t.$$
(b):
Applying the results from (a) and plugging the value $t = 10\text{s}$ yield
$$x(t) = 75\text{m}+(4.1\text{m/s})\cdot 10\text{s}+\frac{1}{2}(2\text{m/s}^2)\cdot(10\text{s})^2 = 216\text{m}$$
and
$$y(t) = -50\text{m}+(-1.1\text{m/s})\cdot 10\text{s}+\frac{1}{2}(-0.54\text{m/s}^2)\cdot(10\text{s})^2 = -88\text{m}.$$
It follows that her position is then given by $r(t) = (216\hat{\imath} - 88\hat{\jmath})\text{m}$. ||

Theorem 2.1: Time of Flight

The time of flight, denoted by T_{tof}, is defined to be $T_{\text{tof}} = \dfrac{2(v_0 \sin(\theta_0)}{g}$.

Proof:

Since
$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = \big(v_0 \sin(\theta_0)\big)t - \frac{1}{2}gt^2 = 0.$$
Rearranging yields
$$t\Big(v_0 \sin(\theta_0) - \frac{gt}{2}\Big) = 0,$$
dividing t from both sides (provided possible) yields
$$v_0 \sin(\theta_0) = \frac{g}{2}t,$$
result follows.

□

Theorem 2.2: Trajectory

The trajectory equation is of the form $y = ax + bx^2$, where $a = \tan(\theta_0)$ and

$$b = -\frac{g}{2\big(v_0 \cos(\theta_0)\big)^2}.$$

Proof:

Take $x_0 = y_0 = 0$ and then $x = v_{0x}t$ yields

$$t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos(\theta_0)},$$

similarly,

$$y = \big(v_0 \sin(\theta_0)\big)\Big(\frac{x}{v_0 \cos(\theta_0)}\Big) - \frac{1}{2}g\Big(\frac{x}{v_0 \cos(\theta_0)}\Big)^2,$$

rearranging yields

$$y = \big(\tan(\theta_0)\big)x - \Big(\frac{g}{2\big(v_0 \cos(\theta_0)\big)^2}\Big)x^2.$$

□

Theorem 2.3: Range

The range, or the horizontal distance traveled by the projectile, is given by

$$R = \frac{v_0^2 \sin(2\theta_0)}{g}.$$

Proof:

One has

$$y = \big(\tan(\theta_0)\big)x - \Big(\frac{g}{2\big(v_0 \cos(\theta_0)\big)^2}\Big)x^2,$$

setting $y = 0$ in this equation yields solutions $x = 0$, corresponding to the lauch point, and

$$x = \frac{2v_0^2 \sin(\theta_0)\cos(\theta_0)}{g},$$

summarizing, one has, since $2\sin(\theta_0)\cos(\theta_0) = \sin(2\theta_0)$, substituting $x = R$ for range, one has the desired result.

□

Example 2.3:

A golfer finds himself in two different situations on different holes. On the second hole he is 120m from the green and wants to hit the ball 90 m and let it run onto the green. He angles the shot low to the ground at 30° to the horizontal to let the ball roll after impact. On the fourth hole he is 90 m from the green and wants to let the ball drop with a minimum amount of rolling after impact. Here, he angles the shot at 70° to the horizontal to minimize rolling after impact. Both shots are hit and impacted on a level surface.

(a) What is the initial speed of the ball at the second hole?

(b) What is the initial speed of the ball at the fourth hole?

(c) Write the trajectory equation for both cases.

Solution:

(a):

We have $R = \dfrac{v_0 \sin(2\theta_0)}{g}$ hence

$$v_0 = \sqrt{\frac{Rg}{\sin(2\theta_0)}} = \sqrt{\frac{90.0\text{m}(9.8\text{m/s}^2)}{\sin\big(2(30^\circ)\big)}} = 31.9\text{m/s}.$$

(b):
Similarly, one has

$$v_0 = \sqrt{\frac{Rg}{\sin(2\theta_0)}} = \sqrt{\frac{90.0\text{m}(9.8\text{m/s}^2)}{\sin\big(2(70^\circ)\big)}} = 37.0\text{m/s}.$$

(c):
The trajectory is given by $y = ax + bx^2$, where $a = \tan(\theta_0)$ and $b = -\dfrac{g}{2\big(v_0 \cos(\theta_0)\big)^2}$. That is,

$$y = x\left(\tan(\theta_0) - \frac{g}{2\big(v_0\cos(\theta_0)\big)^2}x\right),$$

for the second hole, one has

$$y = x\left(\tan(30^\circ) - \frac{9.8\text{m/s}^2}{2\big(31.9\text{m/s}\cos(30^\circ)\big)^2}x\right) = 0.58x - 0.0064x^2,$$

for the fourth hole, one has

$$y = x\left(\tan(70^\circ) - \frac{9.8\text{m/s}^2}{2\big(37.0\text{m/s}\cos(70^\circ)\big)^2}x\right) = 2.75x - 0.0306x^2. \qquad ||$$

2.2 Uniform Circular Motion

In one-dimensional kinematics, objects with a constant speed have zero accelerati-on. However, in multidimensional case, even if the speed is a constant, a particle can still have acceleration if it moves along a curved trajectory such as a circle. In this case the velocity vector is changing, or, $\dfrac{dv}{dt} \neq 0$. As the particle moves counter-clockwise in time Δt on the circular path, its position vector moves from $r(t)$ to $r(t + \Delta t)$. The velocity vector has constant magnitudee and is tangent to the path as it changes from $v(t)$ to $v(t + \Delta t)$, chaning its direction only. Since the velocity vector $v(t)$ is perpendicular to the position vector $r(t)$, the triangles formed by the position vectors and Δr, and the velocity vectors and Δv are similar. Furthermore, since $\|r(t)\| = \|r(t + \Delta t)\|$ and $\|v(t)\| = \|v(t + \Delta t)\|$, the two triangles are isosceles. From these facts we can make the assertion

Theorem 2.4: Centripetal Acceleration

The radical acceleration, denoted by a_c, is defined to be $a_c = \dfrac{v^2}{r}$.

Proof:

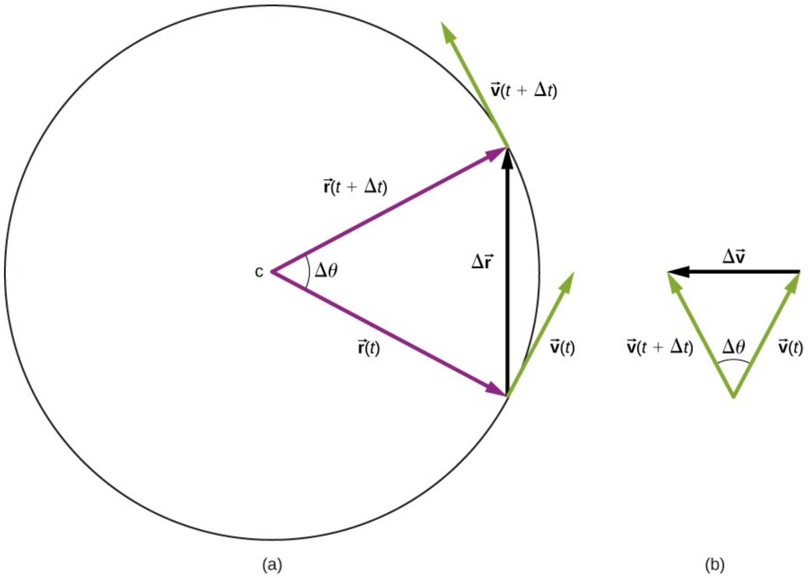
Since $\|r(t)\| = \|r(t+\Delta t)\|$ and $\|v(t)\| = \|v(t+\Delta t)\|$, one has $\frac{\Delta v}{v} = \frac{\Delta r}{r}$, i.e., $\Delta v = \frac{v}{r}\Delta r$. Then according to the formula of the acceleration, we have

$$a := \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r}\left(\lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}\right) = \frac{v^2}{r},$$

since $v := \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$.

□

Note that the direction of the acceleration can also be found by noting that as Δt and therefore $\Delta\theta$ approahches 0, the vector Δv approaches a direction perpendicular to v. In the limit $\Delta t \to 0$, Δv is perpendicular to v. Since v is tangent to the circle, the acceleration $\frac{dv}{dt}$ points toward the center of the circle. Summarizing, a particle moving in a circle at a constant speed has an acceleration with magnitude given by **Theorem 2.4**.



(Figure 2.1)

A particle executing circular motion can be described by its position vector $r(t)$. As the particle moves on the circle, its position vector sweeps out the angle θ with the x -axis. This derives the equations of motion for uniform circular motion.

Definition: Uniform Circular Motion

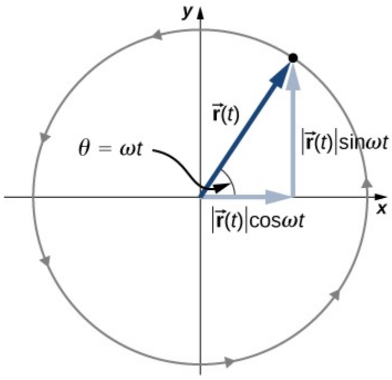
The angular frequency of the particle is denoted by ω, the distance function is given by $r(t) = A\cos(\omega t) \oplus A\sin(\omega t)$.

The angular frequency has units of radians (rad) per second and is simply the number of radians of angular measure through which the particle passes per second. The angle θ that the position vector has at any particular time is ωt. If T is the period of motion, or the time to complete one revolution (i.e., 2π rad), then $\omega = \frac{2\pi}{T}$.

Definition: Velocity and Acceleration

The velocity is given by $v(t) = \frac{dr(t)}{dt} = -A\omega\sin(\omega t) \oplus A\omega\cos(\omega t)$.

The acceleration is given by $a(t) = \dfrac{dv(t)}{dt} = -A\omega^2\cos(\omega t) \oplus -A\omega^2\sin(\omega t)$. In particular, $a(t) = -\omega^2 r(t)$.



(Figure 2.2)

Example 2.4:

A proton has speed 5×10^6m/s and is moving in a circle in the xy-plane of radius $r = 0.175$m. What is its position in the xy-plane at the time $t = 200$ns, where $1\text{ns} = 1 \times 10^{-7}$s?

Solution:

We have

$$T = \frac{2\pi r}{v} = \frac{2\pi(0.175\text{m}}{5.0 \times 10^6\text{m/s}} = 2.20 \times 10^{-7}s,$$

hence

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.20 \times 10^{-7}s} = 2.856 \times 10^7\text{rad/s}.$$

The position of the particle at $t = 2.0 \times 10^{-7}$s with $A = 0.175$m is then

$$r(2.0 \times 10^7\text{s}) = \Big(A\cos(\omega \cdot 2.0 \times 10^{-7}\text{s}) \oplus A\sin(\omega \cdot 2.0 \times 10^{-7}s)\Big)\text{m}$$
$$= (0.147 \oplus -0.095)\text{m}. \qquad ||$$

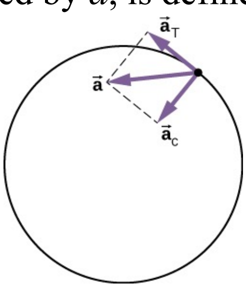
For the nonuniform cicular motion, however, if the speed of the particle is changing, then it has a tangential acceleration that is the time rate of change of the magnitude of the velocity, namely,

Definition: Tangential Acceleration

The tangential acceleration, denoted by a_T, is defined by $a_T = \dfrac{d|v|}{dt}$.

Definition: Total Acceleration

The total acceleration, denoted by a, is defined by $a = a_c + a_T$.



(Figure 2.3)

Example 2.5:

A particle moves in a circle of radius $r = 2.0\text{m}$. During the time interval from $t = 1.5\text{s}$ to $t = 4.0\text{s}$ its speed varies with time according to

$$v(t) = c_1 - \frac{c_2}{t^2},\ c_1 = 4.0\text{m/s, and } c_2 = 6.0m \cdot s.$$

What is the total acceleration of the particle at $t = 2.0\text{s}$?

Solution:

The velocity is given by

$$v(t) = c_1 - \frac{c_2}{t^2} = v(2.0\text{s}) = \left(4.0\text{m/s} - \frac{6.0\text{m}\cdot\text{s}}{(2.0)^2\text{s}}\right) = 2.5\text{m/s},$$

then the centripetal acceleration is given by

$$a_c = \frac{v^2}{r} = \frac{(2.5\text{m/s})^2}{2.0\text{m}} = 3.1\text{m/s}^2,$$

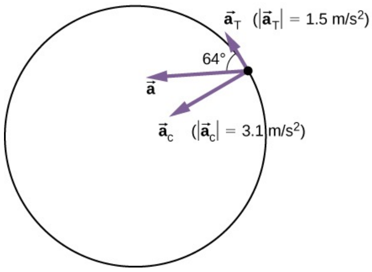
directed toward the center of the circle. Tangential acceleration is

$$a_T = \left|\frac{dv}{dt}\right| = \frac{2c_2}{t^3} = \frac{12.0}{(2.0)^3}\text{m/s}^2 = 1.5\text{m/s}^2.$$

Hence the total acceleration is

$$|a| = \sqrt{3.1^2 + 1.5^2}\text{m/s}^2 = 3.44\text{m/s}^2,$$

and $\theta = \arctan\left(\frac{3.1}{1.5}\right) = 64^\circ$ from the tangent to the circle. ||



(Figure 2.4)

We introduce relative motion in one dimension first, because the velocity vectors simplify to having only two possible directions. Take the example of the person sitting in a train moving east. If we choose east as the positive direction and Earth as the reference frame, then we can write the velocity of the train with respect to the Earth as 10m/s $\hat{i}$ east, where the subscripts TE refer to train and Earth. Let's now say the person gets up out of her seat and walks toward the back of the train at 2m/s. This tells us she has a velocity relative to the reference frame of the train. Since the person is walking west, in the negative direction, we write her velocity with respect to the train as $v_{\text{PT}} = -2\text{m/s}\ \hat{i}$. We can add the two velocity vectors to find the velocity of the person with respect to Earth.

Definition: Relative Velocity

The relative velocity, denoted by v_{PE}, is given by $v_{\text{PE}} = v_{\text{PT}} + v_{\text{TE}}$.

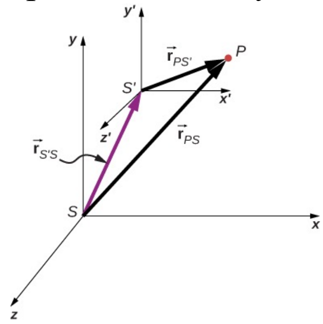
In two-dimensional, we have

Definition: Two-Dimensional Relative Velocity

The two-dimensional relative velocity is given by $r_{\text{PS}} = r_{\text{PS'}} + r_{\text{S'S}}$.

The relative velocities are the time derivatives of the position vectors.

Therefore, $v_{PS} = v_{PS'} + v_{S'S}$. The velocity of a particle relative to S is equal to its velocity relative to S' plus the velocity of S' relative to S.



(Figure 2.5)

Simiarly, the velocity is given by

$$v_{PC} = v_{PA} + v_{AB} + v_{BC}. \tag{2.1}$$

The acceleration is also formulated by

$$a_{PS} = a_{PS'} + a_{S'S}. \tag{2.2}$$

3. Overview on Newton's Laws of Motion

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