Lecture Notes Week 1
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Abstract:
In this chapter we are going to set up the notations and elementary terminologies for later use. We introduce the scope and scale of physics in the first subsection, then for every scope and scale we are able to assign units, and we can also derive a conversion between any units. We proceed the discussion of dimension analysis in the third subsection. Then we introduce the estimates and Fermi Calculations in the fourth part. Lastly we introduce the significant figures, where we can describe the accuracy and errors of an experiment.

Table of Contents:
1.1 The Scope and Scale of Physics
1.2 Units, Standards, and Conversions
1.3 Dimension Analysis
1.4 Estimates and Fermi Calculations
1.5 Significant Figures
Reference

1.1 The Scope and Scale of Physics

Science consists of theories and laws that are the general truths of nature, as well as the body of knowledge they encompass. Scientists are continuously trying to expand this body of knowledge and to perfect the expression of the laws that describe it. Physics, which comes from the Greek phúsis, meaning “nature,” is concerned with describing the interactions of energy, matter, space, and time to uncover the fundamental mechanisms that underlie every phenomenon. This concern for describing the basic phenomena in nature essentially defines the scope of physics.

Definition: Order of Magnitude

The order of magnitude of a number is the power of 10 that most closely approximates it. Thus, the order of magnitude refers to the scale (or size) of a value. Each power of 10 represents a different order of magnitude.

Defintion: Model

A model is a representation of something that is often too difficult (or impossible) to display directly. Although a model is justified by experimental tests, it is only accurate in describing certain aspects of a physical system.

Definition: Theory

For a scientist, a theory is a testable explanation for patterns in nature supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena whereas others do not.

Definition: Law

A law uses concise language to describe a generalized pattern in nature supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation.

Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence.

Before we dive any deeper. We should be familiar in the numbers we are using. We shall use the decimals hence we shall use 10 as a basis. For any number smaller than 10, we use the number itself, for any number greater than 10, especially for those having digits, e.g., 12345, where we can then use 1.2345×10^4 to represent this number. Moreover, there is a matter of syntax, i.e., instead of writing 12345 directly, we shall denote it by 12,345. This reads "12 thousands 3 hundreds and 45", since we choose the scale to be in the ascending form: thousand, million, billion, to denote the value 10^3,10^6, and 10^9, respecitvely, then it makes sense to use a comma to separate every 3 consequtive digits.

1.2 Units, Standards, and Conventions

Recall that in basic calculus where we encountered the evaluation of a series. In particular, if we are going to evaluate the series $\sum_{i=1}^{n} \frac{a^i}{\pi}$, we know it is equivalent to consider the equation

$$\frac{a}{\pi} + \frac{a^2}{\pi} + \cdots + \frac{a^n}{\pi}. \tag{1.1}$$

According to the summation of a geometric sequence we know that this is equivalent to the value $\frac{a}{\pi} \frac{(1-a^n)}{(1-a)}$.

The calculation is valid since we know that the symbol $\sum$ means, and we know which elements are going to be fixed, and which elements are going to change. Therefore becore characterizing the space we are working with. We need to be familiar with the elements of the space, in particular the units we are using to describe them.

There are two major systems of units are used in the world: **SI units** (for the French Système International d'Unités), also known as the metric system, and **English units** (also known as the customary or imperial system). English units were historically used in nations once ruled by the British Empire and are still widely used in the United States. English units may also be referred to as the foot–pound–second (fps) system, as opposed to the centimeter–gram–second (cgs) system. You may also encounter the term SAE units, named after the Society of Automotive Engineers. Products such as fasteners and automotive tools (for example, wrenches) that are measured in inches rather than metric units are referred to as SAE fasteners or SAE wrenches.

In any system of units, the units for some physical quantities must be defined through a measurement process. These are called the **base quantities** for that system and their units are the system's **base units**. All other physical quantities can then be expressed as algebraic combinations of the base quantities. Each of these physical quantities is then known as a **derived quantity** and each unit is called a **derived unit**. The choice of base quantities is somewhat arbitrary, as long as they are independent of each other and all other quantities can be derived from them. Typically, the goal is to choose physical quantities that can be measured accurately to a high precision as the base quantities. The reason for this is simple. Since the derived units can be expressed as algebraic combinations of the base units, they can only be as accurate and precise as the base units from which they are derived.

In mechanics, we need to define only three units:

(i)	Unit of Length:	Meter (m)
(ii)	Unit of Mass:	Kilogram (kg)
(iii)	Unit of Time:	Second (s)
(iv)	Unit of Electric Current:	Ampere (A)
(v)	Unit of Thermodynamic temperature:	Kelvin (K)
(vi)	Amount of substance:	Mole (mol)

We now give detailed descriptions of (i) (ii), and (iii).

Meter:

The meter is the length equal to the distance traveled by light in vacuum, in a time of $\frac{1}{299{,}792{,}458}$. Note that in specific, the velocity of light is

$$c = 2.99792458 \times 10^8 m/s.$$

Indeed, we know the velocity measures the rate of change of an object's position with respect to time. Therefore we see that, for a distance function d, the velocity is defined to be the ratio $v := \frac{d}{t}$, it follows that we use the $\frac{m}{s}$, where m is the unit of length and s is the unit of time, to represent this relation. In fact, if we ever encounter some derived units that we have never seen before, we can use the units to make a claim of how this value is obtained. This is the motivation of defining the units, since then we can use the units to derive anything.

Kilogram:

The SI unit for mass is the kilogram (abbreviated kg); From 1795–2018 it was defined to be the mass of a platinum–iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. However, this cylinder has lost roughly 50 micrograms since it was created. Because this is the standard, this has shifted how we defined a kilogram.

Second:

A cesium (-beam) atomic clock (or cesium-beam frequency standard) is a device that uses as a reference the exact frequency of the microwave spectral line emitted by atoms of the metallic element cesium, in particular its isotope of atomic weight 133 (“Cs-133”). The frequency, $f_0 = 9{,}192{,}631{,}770$ hertz (Hz := cycles/second), provides the fundamental unit of time, which may thus be measureed by cesium clocks.

Period: $$T_0 = \frac{1}{f_0} = \frac{1}{9{,}192{,}631{,}770} s = 1.0878 \times 10^{-11} s = 0.11 ns.$$

One second is the duration of 9,192,631,770 periods.

The time measurement accuracy is 2 nanoseconds per day or one second in 1,400,000 years. It is the most accurate realization of a unit that mankind has yet achieved. That is, 0.1 ns/day = 3.65 x 10-8 sec/1 year = 1 sec/27.4 million years. One can check details in [1].

SI units are part of the metric system, which is convenient for scientific and engineering calculations because the units are categorized by factors of 10. For exam-ple, a centimeter is one-hundredth of a meter (in symbols, 1 cm = 10^{-2} m) and a kilometer is a thousand meters (1 km = 10^3 m). Similarly, a megagram is a million grams (1 Mg = 10^6 g), a nanosecond is a billionth of a second (1 ns = 10^{-9} s), and a terameter is a trillion meters (1 Tm = 10^{12} m).

There are two kinds of SI units; (a) the SI based units, and (b) the SI derived units

The SI base units:

Meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), and candela (cd).

The SI derived units Units:

For other quantities, such as velocity, acceleration, force, energy, and power, are derived from these basic units.

Area: m^2

Volume: m^3

Velocity: m/s …

It is natural to introduce the conversion between different units in order to provide a well-defined operation.

We sometimes denote the units of length to be inch. The relation between meter a-nd inch is that 1 m is equivalent to 1.0936 yards, or 39.370 inches. Similarly, sometimes we denote the units of mass by pounds. The relation between kilogram and pounds is that 1 pound (lb) is equal to 0.45359237 kilograms (kg).

Exercise 1.1:

The density of iron is 7.86 g/cm^3 under standard conditions. Convert this to kg/m^3.

Solution:

We need to convert grams to kilograms and cubic centimeters to cubic meters.

The conversion factors we need are and However, we are dealing with cubic centimeters. One has

$$7.86\frac{g}{cm^3}\times\frac{kg}{10^3 g}\times\left(\frac{cm}{10^{-2}m}\right)^3 = 7.86\times 10^3 kg/m^3. \qquad ||$$

We sometimes encounter data in units other than those used in SI system. In this case we need to convert the units to the SI system, using conversion factors. For example, we consider a speed of 65 miles/hour. 1 mile = 1610 m. 1 hour = 60 min = 60 x 60=3600 s.

1.3 Dimension Analysis

We start with the definition

Defintion: Dimension

The dimension of any physical quantity expresses its dependence on the base quantities as a product of symbols (or powers of symbols) representing the base quantities.

The three fundamental dimensions are length L, mass M and time T. To give the dimension of a variable we use square brackets. For example, if x is some distance then it is dimensionally a length, and we write: [x] = L. A velocity is length per time, so if v is a velocity we would write [v] = L/T.

If two quantities are equal they must be dimensionally equal. Moreover, as illustrated previously, if we add two quantities they must be dimensionally the same.

Base Quantity	Symbol for Dimension
Length	L
Mass	M
Time	T
Current	I
Thermodynamic Temperature	Θ
Amount of Substance	N

We see that if $\alpha + \beta = \gamma$ is a correct expression then it must follow that the three variables have the same dimension, i.e., $\alpha + \beta = \gamma \Rightarrow [\alpha] + [\beta] = [\gamma]$. It is also the case that the dimension of the product of two variables is the product of their dimensions: $\alpha\beta = [\alpha][\beta]$.

On the other hand, a dimensionless quantity is some number, like π, 2, and $\sqrt{3}$, that would be expressed without units. If κ is dimensionless we denote $[\kappa] = 1$, since multiplying by a dimensionless quantity doesn't change the dimension of a quantity.

The condition that our expressions must be dimensionally correct puts a constraint on the final form of our expressions. In fact, in many cases dimensional analysis will uniquely determine our expressions up to multiplicative dimensionless constants. Suppose one is trying to recall an expression for the surface area of a sphere in terms of its radius. An area A is dimensionally a length squared and a radius r is a length. It follows that the expression for the area in terms of the radius must have the general form:

$$[\mathrm{A}] = L^2 \text{ and } [\mathrm{r}] = L \Rightarrow A = \kappa r^2 \text{ where } [\kappa] = 1.$$

In the case the dimensionless constant is $\kappa = 4\pi$.

Exercise 1.2:

As an example of this suppose we want to find an expression for a distance [x] = L in terms of an acceleration [a] = L/T^2 and a speed [v] = L/T. We can choose a general form of this expression as: $x = \kappa a^m v^n$ where $\kappa = [1]$.

Solution:

We can solve for *m* and *n* using dimensional analysis by insisting the expression is dimensionally correct:

$$[x] = [\kappa][a]^m[v]^n \Rightarrow L = 1 \cdot \left(\frac{L}{T^2}\right)^m \left(\frac{L}{T}\right)^n = 1 \cdot \frac{L^m}{T^{2m}} \frac{L^n}{T^n}.$$

Then we have

$$\begin{cases} 2m + n = 0 \\ m + n = 1 \end{cases},$$

solving the linear equation yields $m = -1$ and $n = 2$ so we can conclude that

$$x = \kappa \frac{v^2}{a} \text{ where } [\kappa] = 1. \qquad ||$$

One further point that needs to be mentioned is the effect of the operations of calculus on dimensions. We have seen that dimensions obey the rules of algebra, just like units, but what happens when we take the derivative of one physical quantity with respect to another or integrate a physical quantity over another? The derivative of a function is just the slope of the line tangent to its graph and slopes are ratios, so for physical quantities v and t, we have that the dimension of the derivative of v with respect to t is just the ratio of the dimension of v over that of t:

$$[\frac{dv}{dt}] = \frac{[v]}{[t]} \tag{1.2}$$

Similarly, since integrals are just sums of products, the dimension of the integral of v with respect to t is simply the dimension of v times the dimension of t:

$$\int vdt = [v] \cdot [t]. \tag{1.3}$$

Moreover, we have seen that the dimension if closed under addition, but the formation of multiplication yields another dimension, e.g. $10m \times 10m = 100m^2$, which is usually to measure the area of a surface.

1.4 Estimates and Fermi Calculations

On many occasions, physicists, other scientists, and engineers need to make estimates for a particular quantity. Other terms sometimes used are guesstimates, order-of-magnitude approximations, back-of-the- envelope calculations, or Fermi calculations. (The physicist Enrico Fermi mentioned earlier was famous for his ability to estimate various kinds of data with surprising precision.) Will that piece of equipment fit in the back of the car or do we need to rent a truck? How long will this download take? About how large a current will there be in this circuit when it is turned on? How many houses could a proposed power plant actually power if it is built? Note that estimating does not mean guessing a number or a formula at random.

Definition: Estimation

> Estimation means using prior experience and sound physical reasoning to arrive at a rough idea of a quantity's value.

Because the process of determining a reliable approximation usually involves the identification of correct physical principles and a good guess about the relevant variables, estimating is very useful in developing physical intuition. Estimates also allow us to perform "sanity checks" on calculations or policy proposals by helping us rule out certain scenarios or unrealistic numbers. They allow us to challenge others (as well as ourselves) in our efforts to learn truths about the world.

Exercise 1.3:

> Estimate the total mass of the oceans on Earth.

Solution:

> We know the density of water is about 10^3 kg/m^3, we can estimate the volume of the oceans as surface area times average depth, or V = AD; we can approximate Earth as a sphere and use the formula for the surface area of a sphere of diameter d by $A = \pi d^2$; we take the average depth to be around $3 \times 10^3 m$.

We estimate the surface area of Earth (and hence the surface area of Earth's oceans) to be roughly

$$A = \pi d^2 = \pi(10^7 m)^2 \approx 3 \times 10^{14} m^2.$$

Next, using our average depth estimate of $D = 3 \times 10^3 m$ which was obtained by bounding, we estimate the volume of Earth's oceans to be

$$V = AD = (3 \times 10^{14} m^2)(3 \times 10^3 m) = 9 \times 10^{17} m^3.$$

Last, we estimate the mass of the world's oceans to be

$$M = \rho V = (10^3 kg/m^3)(9 \times 10^{17} m^3) = 9 \times 10^{20} kg.$$

Thus, we estimate that the order of magnitude of the mass of the planet's oceans is approximately $10^{21} kg$. ||

1.5 Significant Figures

Science is based on observation and experiment — that is, on measurements. We therefore define some terminology to measure the behavior.

Definition: Accuracy

Accuracy is how close a measurement is to the accepted reference value for that measurement.

For example, let's say we want to measure the length of standard printer paper. The packaging in which we purchased the paper states that it is 11.0 in. long. We then measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the reference value of 11.0 in. In contrast, if we had obtained a measurement of 12 in., our measurement would not be very accurate. Notice that the concept of accuracy requires that an accepted reference value be given.

Definition: Precision

The precision of measurements refers to how close the agreement is between repeated independent measurements (which are repeated under the same conditions).

The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements is to determine the range, or difference, between the lowest and the highest measured values. In this case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by, at most, 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9 in., 11.1 in., and 11.9 in., then the measurements would not be very precise because there would be significant variation from one measurement to another. Notice that the concept of precision depends only on the actual measurements acquired and does not depend on an accepted reference value.

The precision of a measuring system is related to the **uncertainty** in the measurements whereas the accuracy is related to the **discrepancy** from the accepted reference value. Uncertainty is a quantitative measure of how much your measured values deviate from one another. There are many different methods of calculating uncertainty, each of which is appropriate to different situations. Some examples include taking the range (that is, the biggest less the smallest) or finding the standard deviation of the measurements. Discrepancy (or "measurement error") is the difference between the measured value and a given standard or expected value. If the measurements are not very precise, then the uncertainty of the values is high. If the measurements are not very accurate, then the discrepancy of the values is high.

Another method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty δA, the percent uncertainty is defined as

$$\text{Percent Uncertainty } = \frac{\delta A}{A} \times 100\,\%\,.$$

Exercise 1.4:

A grocery store sells 5-lb bags of apples. Let's say we purchase four bags during the course of a month and weigh the bags each time. We obtain the following measurements:

Week 1 weight: 4.8 lb
Week 2 weight: 5.3 lb
Week 3 weight: 4.9 lb
Week 4 weight: 5.4 lb

We then determine the average weight of the 5-lb bag of apples is 5.1 ± 0.3 lb from using half of the range. What is the percent uncertainty of the bag's weight?

Solution:

Substitute the values into the equation:

$$\text{Percent Uncertainty } = \frac{\delta A}{A} \times 100\,\% = \frac{0.3lb}{5.1lb} \times 100\,\% = 5.9\,\% \approx 6\,\%\,. \quad \|$$

Reference:

[1]: NIST-F1 Cesium Fountain Atomic Clock. The Primary Time and Frequency Standard for the United States http://tf.nist.gov/cesium/fountain.html.