All types of Convergence

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Abstract:
In order for convenience in checking definition along learning in Math, I found it necessary to gather definitions of the same kind so that distinguishing the differences among them will no longer be a tedious job. This work is very hard to do even many materials are available, if you want to use for commercial use, please contact me for permission. Furthermore, the work is still long from finished, if you want to contribute for more or better definitions, please contact me.

1.
Definition: converge(in measure)

A sequence $\{f_n\}$ of almost everywhere finite valued, measurable functions converge in measure to the measurable function f if,

$$\forall \epsilon > 0, \lim_n \mu(\{x; |f_n(x) - f(x)| \geq \epsilon\}) = 0.$$

2.
Definition: pointwise converge

Given a sequence $f_n : X \longrightarrow Y$ countable or not, we say that the sequence $\{f_n\}$ converges pointwise to the function $f : X \longrightarrow Y$ if $\lim_{n\to\infty} f_n(x) = f(x) \forall x \in X$.

3.
Definition: uniformly converge

The sequence $\{f_n\}$ of real-valued functions is said to uniformly converge to a function f, denoted as $\{f_n\} \to f$ uniformly almost everywhere, if there exists a set E_0 with measure 0 such that $\forall \epsilon > 0$, $\exists n_0 = n_0(\epsilon) \in \mathbb{Z}$, such that

$$|f_n(x) - f(x)| < \epsilon \forall n \geq n_0 \forall x \notin E_0.$$

4.
Definition: almost uniform converge

A sequence $\{f_n\}$ of almost everywhere finite valued measurable functions will be said to converge to the measurable function f almost uniformly if $\forall \epsilon > 0$, there exists a measurable set F such that $\mu(F) < \epsilon$ and such that the sequence $\{f_n\}$ converges to f uniformly on F^c.

5.
Definition: weak converge

$\{x_n\}$ is a sequence in X, $x_n \in X \; \forall n$, $x_n \rightharpoonup x$ weakly if $\forall l \in X^*$, $l(x_n) \to l(x)$.

6.

Definition: converges in the mean/mean converge

A sequence $\{f_n\}$ of integrable functions converges in the mean, or mean converges, to an integrable function f, if $\rho(f_n, f) = \int |f_n - f| \, d\mu \to 0$ for sufficiently large n.